Measurement of the velocity of light

Aims

- To measure the velocity of light in air by measuring a phase shift $\Delta\varphi$ of a periodic light signal in response to a change in propagation distance Δs.
- To measure the velocity of light in water and Perspex and thereby determine the refractive indices for these substances.

Experimental Background

Consider a source that emits light with a periodic time dependent intensity of the form

$$I(t) = I_0 \cos(2\pi f t)\,. \qquad [1]$$

If this signal is detected by a receiver placed at a distance Δs from the source, then there will be a time delay

$$\Delta t = \frac{\Delta s}{c} \qquad [2]$$

Which corresponds to a phase difference $\Delta\varphi$ between the emitted and detected waveforms, where

$$\Delta\varphi = 2\pi f \Delta t = \frac{2\pi \Delta t}{T} \qquad [3]$$

and T is the time period of the signal. Thus, the detected signal will be of the form

$$U(t) = A\cos(2\pi f t - \Delta\varphi) \qquad [4]$$

Measurement of the phase shift $\Delta\varphi$ between the emitted and detected waveforms for light traveling a distance Δs then allows the speed of light c to be determined from

$$c = \frac{2\pi f \Delta s}{\Delta\varphi} \qquad [5]$$

The main drawback of this method is that for typical path lengths possible in a crowded laboratory, very high frequencies are required to produce measurable phase shifts. This requires the use of specialized and expensive oscilloscopes to display the waveforms. The system used here, however, uses a mixing technique which allows the phase shift between high frequency signals to be observed on a standard laboratory oscilloscope.

In this experiment, the frequency of the emitted light is f_1 = 60MHz. The 60 MHz signal detected at the receiver is mixed (multiplied) electronically with a slightly lower frequency signal with f_2 = 59.9 MHz. From the cosine addition formula

$$\cos\theta\cos\varphi = \frac{1}{2}\left(\cos(\theta+\varphi) + \cos(\theta-\varphi)\right) \qquad [6]$$

the mixed signal at the receiver may be written as

$$U(t) = A\cos(2\pi f_1 t - \Delta\varphi)\cos(2\pi f_2 t) = \frac{1}{2}A(\cos(2\pi t(f_1 + f_2) - \Delta\varphi) + \cos(2\pi t(f_1 - f_2) - \Delta\varphi))$$

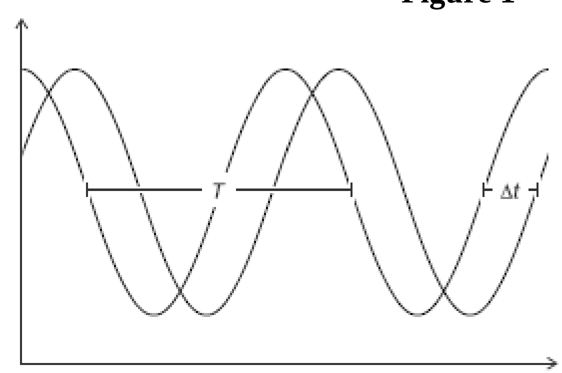
The high frequency ($f_1 + f_2$) term above is easily removed by means of a low pass filter, leaving only the frequency difference term, ie

$$U(t) = \frac{A}{2}\cos(2\pi t(f_1 - f_2) - \Delta\varphi) \qquad [7]$$

Since ($f_1 - f_2$) is only 100 kHz, this signal can be displayed on a standard, low frequency oscilloscope.

For determination of the phase shift, a 60MHz reference signal is provided, which is synchronized with the intensity of the light transmitter, mixed with the same 59.9MHz signal and filtered in the same way as the receiver signal. By displaying both the reference and the mixed receiver signal on the oscilloscope, $\Delta\varphi$ can be measured.

Figure 1



If the time period of these 100kHz signals is T_m and the measured shift on the oscilloscope time base is Δt_m, then

$$\Delta\varphi = \frac{2\pi\Delta t_m}{T_m} \qquad [8]$$

We must remember, however, that the *actual* propagation time of the light signal along the path Δs is *not* Δt_m, but rather

$$\Delta t = \frac{\Delta t_m T_1}{T_m} = \frac{\Delta t_m}{T_m f_1} \qquad [9]$$

where f_1 = 60MHz and T_1 = 16.67ns are the frequency and time period respectively of the original light signal.

The experimental value for the speed of light is thus given by

$$c = \frac{\Delta s}{\Delta t} = \frac{\Delta s T_m f_1}{\Delta t_m} \qquad [10]$$

Because signal delays in connecting leads and circuitry introduce unpredictable phase shifts, a “phase adjust” control is provided, which allows $\Delta\varphi$ to be set to zero for the initial path length between transmitter and receiver.

The apparatus used for this experiment is illustrated in Figures 2 & 3.

Experimental Method

Experiment 1: Speed of light in air

Make sure that the light transmitter/receiver unit is switched on as early as possible; it needs to warm up for around 30 minutes before measurement can commence. During this time you will need to align the equipment. First, set the transmitter unit approximately 1m from the receiver unit and connect it to the receiver unit with the long coaxial cable, align the red spot on the receiver and adjust the insert (e) relative to the collimator (d) to get an even distribution of light. The grub screws (f) can be used to align the spot Connect output “C” of the receiver to channel 2 of the oscilloscope, set the coupling to AC and the time base to 2µs.

Decrease the transmitter-receiver separation to 0.5m, using the osciloscope trace to help with alignment. Connect output “B” of the receiver to channel 1 of the oscilloscope, (set for dual display), with the coupling for both channels set to AC and the trigger set to channel 1. Now adjust the traces so they are symmetric with respect to the horizontal centre line of the screen, also adjust the traces so that the maxima of both lines are on the same horizontal line. Using the phase shift control on the receiver adjust the positions of the traces so that $\Delta\varphi = 0$, and determine the time period T_m.

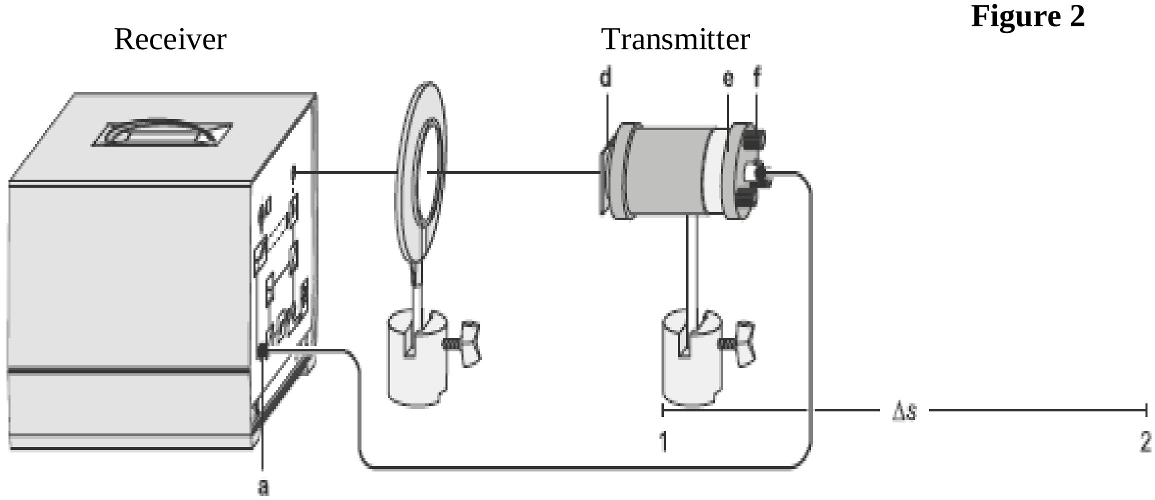


Figure 2

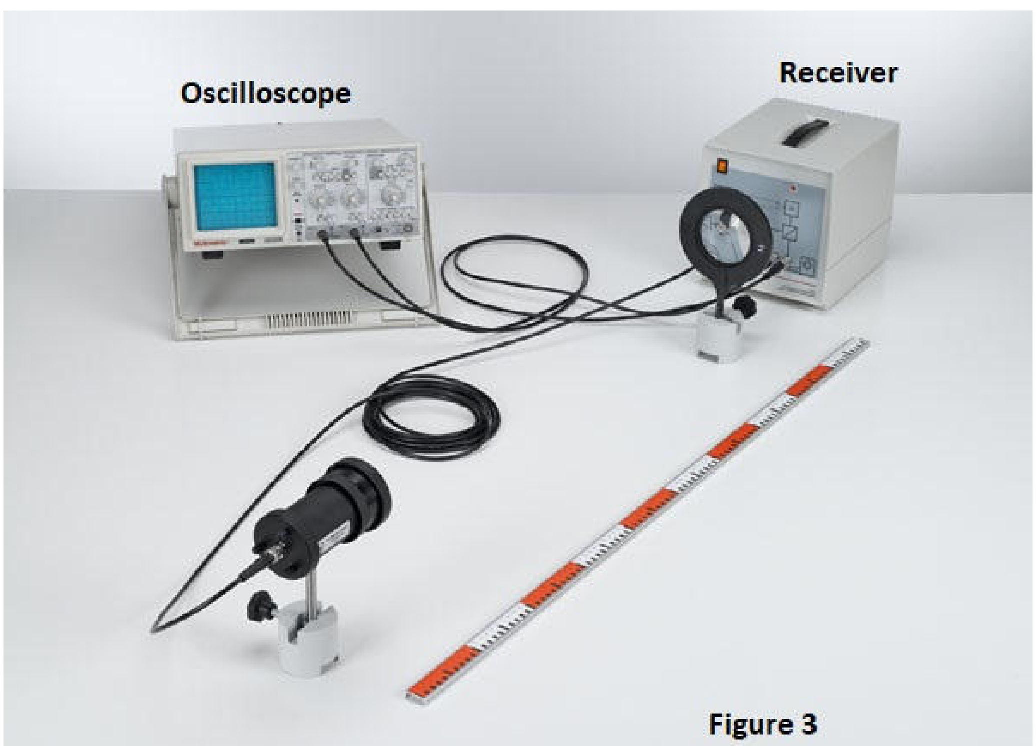


Figure 3

Taking the current position as the zero of displacement, move the transmitter unit a further 0.5m away from the receiver. You should see that the 2 traces are now no longer in phase. Using the oscilloscope set to a time base of 2μs, make sure that the lines are symmetric about the centre line of the screen. Then change the time base of the oscilloscope to either 1 μs or 0.5 μs; this will allow you to make an accurate determination of the apparent propagation time Δt_m.

Now increase the displacement from the receiver in increments of 0.1m up to ≈1.9m and record the apparent propagation time for each of these distances. Plot your data in a suitable form and use the LINEST function in Excel to give a value for the speed of light in air together with associated experimental error.

Experiment 2: Speed of light in perspex and water

When light travels through a medium of refractive index *n,* the speed of light in the medium is given by

$$c_n = \frac{c_0}{n} \qquad [11]$$

where c_0 is the speed of light in a vacuum (assumed, for the purposes of this experiment to be identical to the speed of light in air). If light travels a distance *d* through the medium, the propagation time is given by

$$t_n = \frac{d}{c_n} \qquad [12]$$

whereas the propagation time for the same path length in air is

$$t_0 = \frac{d}{c_0} \qquad [13]$$

QUESTION

Show that

$$c_n = \frac{c_0}{1 + \frac{\Delta t c_0}{d}} \qquad [14]$$

where $\Delta t = (t_n - t_0)$

In this experiment, you will measure the change in propagation time when a medium of a certain refractive index and length d is introduced into the optical path, using similar methods to those used in Experiment 1.

Move the transmitter unit far enough away from the receiver unit and lens so that the long black tube of water can be placed on the optical bench. Make sure that the reference signal and the receiver signal are in phase by using the phase shift control on the receiver as before. Introduce the black tube of water into the beam of light, and with the time base set to a suitable level measure the change in the apparent propagation time Δt_m. The path length, d=1m.

Repeat the procedure with the Perspex cylinder, path length, d=0.3m.

Use equation [14] to determine the speed of light in Perspex and water, and obtain values for the refractive indices of these materials. Recall that the actual propagation time Δt is related to the apparent propagation time Δt_m by equation [9].